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DOI:
10.1063/1.4939901

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(Dated: 20 February 2016)

In order to transfer a range of important optical experiments into the microwave regime, a pair of near-identical, weakly coupled resonators is required. We describe a simple tuning mechanism for taking a pair of coupled, coplanar resonators through the avoided crossing in a controlled way. We see no obvious degradation of their high quality factor and find very good agreement with theoretical expectations.

PACS numbers: 72.25.-g, 85.25.-j, 84.40.Dc, 85.85.+j

I. INTRODUCTION

Thin film superconducting, coplanar microwave resonators (CMRs) have been studied extensively for a wide range of microwave applications, for coupling with vibrating bars and more recently, thin vibrating membranes [1–5]. This latter work has been driven, in part, by their potential for measurement devices such as ultrasensitive detectors[6] and also by a wider interest in the quantum properties of macroscopic objects[7,8] and their coupling to electromagnetic fields.

The present work has been motivated by a suggestion that, instead of a single CMR, a pair of weakly coupled, near identical CMRs could be used, where the splitting frequency $f_{\mathrm{splitting}}$ at the avoided crossing is designed to be close to the mechanical resonant frequency f_{m}[9]. This would closely parallel experiments in optics where two, near identical, optical cavities are separated by a weakly transparent vibrating membrane, and would enable a range of important optical experiments to be transferred to the microwave regime. Despite the weaker coupling strength of the electromagnetic field to mechanical motion, microwave resonators can easily be cooled to temperatures where they behave quantum mechanically. They can also be driven non-linear[10,11] more easily than in optical experiments, leading to a range of more complex behaviours. In practice this requires that the fundamental frequencies of two resonators with high quality factor (Q) are matched to within a few MHz, more than 3 orders of magnitude smaller than a typical CMR frequency of 6 GHz. It is very difficult to match the two CMR frequencies to this degree of accuracy or to obtain reproducibility on this scale, even for two resonators on the same substrate. It is therefore necessary, not only to have weak coupling, but also be able to adjust the frequency of one of the CMRs in situ.

There have been many reports of systems to adjust the frequency of a *single* CMR in situ, utilising magnetic fields [12], ferroelectric layers[13], ferromagnetic layers[14] and in situ SQUIDs[15]. The last three invariably compromise the Q of the CMR, which is undesirable for the proposed experiments. Magnetic fields produce only a few MHz frequency change. Mechanical methods involving voltage controlled dielectric 'paddles' are difficult to fabricate [16] and those based on moving a superconducting pin towards and away from the CMR using piezoelectric motors[17] are expensive and require considerable electromagnetic shielding if they are to be operated at very low temperatures. We report here a mechanical system that is simple, easy to construct and is effective in tuning two coupled resonators through the avoided crossing without observable degradation of the high Q, and find very good agreement with simulations. Such tuning has not previously been demonstrated for *coupled* coplanar resonators. Our system would be easily adaptable as a test bed for a range of important experiments[18–20].

II. EXPERIMENTAL METHODS

A. Design and fabrication of coupled resonator samples

The behaviour of coupled resonances is very well known in mechanical, electrical and quantum systems. Our measurement system (Fig. 1) comprises a pair of thin film niobium $\lambda/2$ microwave resonators in the coplanar waveguide geometry, weakly coupled to each other by a capacitance C_{k} that determines the splitting. They are connected to the input and output ports by small capacitances C_{c} so that coupling to the environment is weak. The devices were made on high resistivity silicon substrates with a 400 nm oxide layer onto which 200 nm of high purity niobium has been sputter deposited[21]. Samples were patterned using conventional photolithography to produce two nominally identical CMRs. Capacitances C_{c} and C_{k} were modelled using the electromagnetic simulator COMSOL[22]. The overlap length l_{c} and the separation Δx were adjusted to give the capacitances for optimal coupling to the environment and the desired splitting. The theoretical resonant frequencies close to the avoided crossing of high Q resonators are shown schematically in Fig. 1c as a function of the inverse inductance per unit length of resonator 1. The devices were mounted onto copper clad, microwave printed circuit board, sol-

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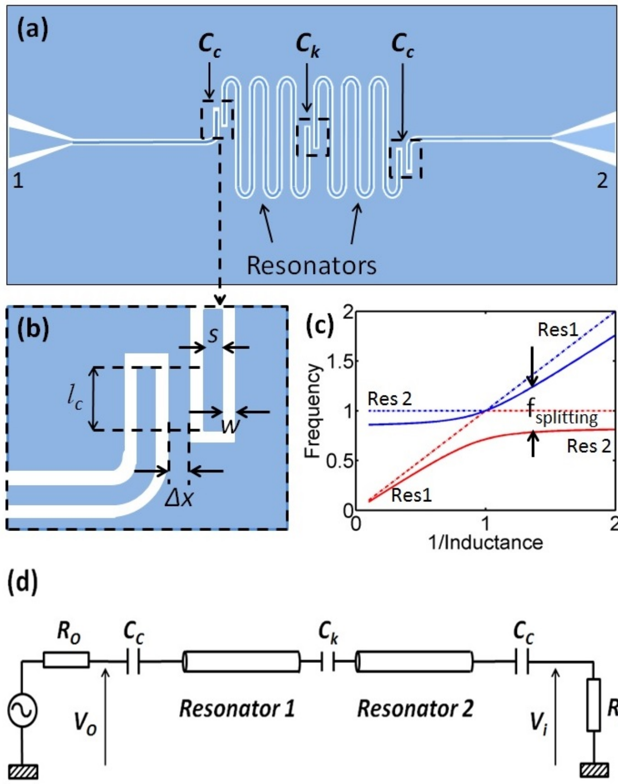

FIG. 1. (a) Schematic layout of the coupled resonators. The solid colour represents the niobium film and the clear sections show the silicon substrate.The substrate is 10×5 mm^2. (b) Details of the coupling capacitors. The centre track width, s= 10 μm and gap, w= 5 μm, give an impedance of close to 50 Ω. The capacitances between the two centre tracks were computed from the overlap lengths l_c and the gap Δx, giving $C_c = 8 \times 10^{-16}$ F. C_k was varied between 1.09×10^{-16} F and 1.57×10^{-15} F for the five different samples. (c) Theoretical upper and lower frequency branches. (d) Equivalent circuit of the coupled resonators used in the theoretical analysis. R_0 = 50 Ω are the input and output impedances of the network analyser. V_i and V_0 are the corresponding input and output voltages.

dered into a copper cavity housing the microwave ports. The experiments were conducted in a liquid helium cryostat with a base temperature of $\sim$ 1.3 K. Thermal noise is unimportant at this temperature and at the microwave levels we are using. The transmission and reflection coefficients were measured with an HP 8720D network analyser. Fig. 2 shows the S_{21} and S_{11} data for sample 4. There is clear evidence that the two resonators are not identical and therefore that measurements are not being taken at the avoided crossing. Firstly, the measured splitting of 9.7 MHz is higher than our calculated value of 6.1 MHz but, more significantly, we see that S_{11} shows very different depths for the lower and higher frequency resonances. This may reflect a variation in the patterning along the length of the chip. Simulations show that a sample tilt of 0.05° relative to the copper ground plane

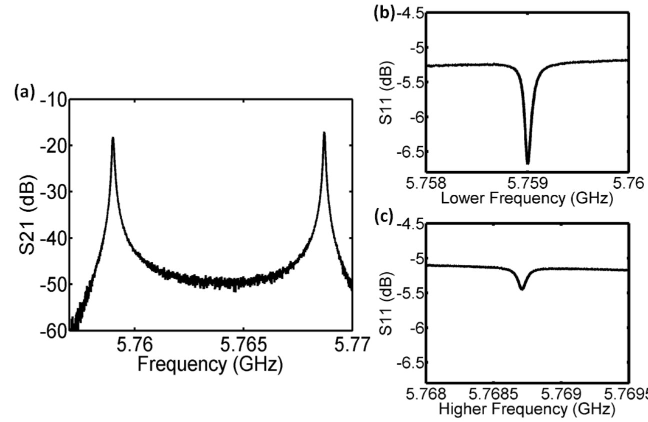

FIG. 2. Magnitudes of transmission S_{21} and reflection S_{11} coefficients for sample 4: (a) S_{21} data showing a splitting of 9.7 MHz. (b) & (c) show the S_{11} data for the lower and upper frequency resonances respectively. The deeper S_{11} identifies the resonator nearest to the port being measured. The effect of any frequency dependence of the cable impedance, both inside and outside the cryostat, was calibrated out by shorting them together at the bottom of the cryostat before the start of each experimental run. The drive levels were adjusted to ensure they were below a level at which non-linearity due to kinetic inductance effects set in.

along the length of the chip would change the capacitance per unit length by a sufficient amount, as would 100 nm of dielectric contamination on one or other of the resonators (assumed $\epsilon_r = 3$). Other samples have shown deviations up to 25 MHz. Whatever the cause, in situ tuning is clearly required for the sort of experiments we are proposing.

B. Design of tuning mechanism

Fig. 3 shows the main features of the tuning mechanism, which is built into the lid of the microwave cavity. The tuning disc is 1.8 mm diameter and is cut from the same stock as the niobium on silicon substrate. The niobium faces down towards the chip and is sited above one of the resonators. As the disc is lowered towards the resonator, it reduces its inductance per unit length and raises its resonant frequency, while having negligible effect on the other resonator. This dominates any capacitive effect due to the dielectric constant of the tuning disc. The probe that holds the disc is made of the machinable insulator, Macor[23]. Separate tests have shown that it has no observable effect on S_{21} of the empty cavity. The macor probe passes through close sliding fits in the brass guide and cavity lid.

The height of the tuning disc is governed by the lever, which acts on the copper bush to raise or lower the macor probe against the force of leaf springs A and B. The lever is pressed by spring C against the conical end of the tuning rod, and is operated by the advance of the

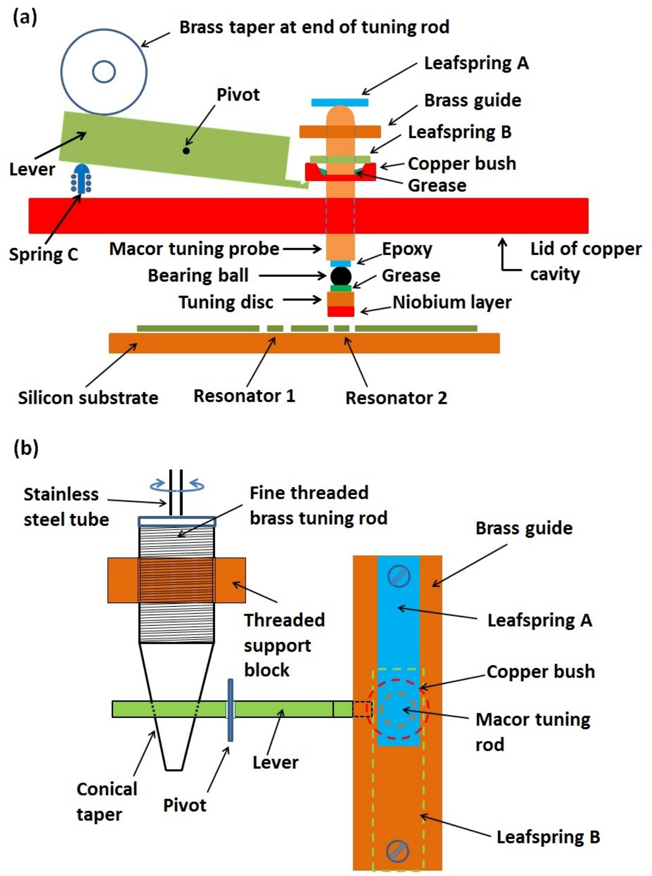

FIG. 3. Schematic showing principle of operation. The two figures are on different scales. (a) shows a side elevation and (b) shows a plan view. The threaded support block for the tuning rod, shown in (b), is bolted firmly to the top of the cavity lid. So also are the supports for the pivot, the brass guide for the macor tuning rod and the outer end of each leaf spring (supports not shown for clarity). The mechanism can be made to act on Resonator 1 simply by rotating the copper cavity under the lid. See text for further details of design and operation.

2° conical taper as the threaded part of the rod is rotated in the 0.635 mm pitch screw thread of the support block. Rotation is achieved using a thin-walled stainless steel tube as a shaft, which passes through an O-ring seal at the top of the cryostat. A special feature of our design is the mechanism for ensuring that the tuning disc is closely parallel to the plane of the resonator. The height scale over which tuning is expected to be effective is the width of the track, which is 10 μm, and this is confirmed by simulations. A tilt angle for the disc of only one degree corresponds to 32 μm across the 1.8 mm width of the disc. Tilt is also undesirable because of the potential for damage to the resonator when the tuning disc is lowered onto it. We minimise these problems by using a small bearing-ball between the macor probe and the tuning disc. The top of the ball is glued firmly to the macor and the tuning disc is attached to the bottom of the ball by a small spot of silicone grease. Prior to cooling, the tuning disc is pushed gently down onto the resonator by the leaf spring A. As the apparatus is cooled, the grease solidifies and parallelism is maintained. The lever acts on the macor probe via the copper bush, whose position on the probe is also fixed by the solidification of grease. The function of leaf spring B is to hold the copper bush firmly against the end of the tuning lever. Prior to cooling, the brass tuning rod is set to around the centre of its travel. This ensures a mid-range starting point for motion of the lever when the grease has solidified.

III. RESULTS

A. Measured splitting in frequency

We are not able to plot the frequencies of the coupled resonators in the form shown in Fig. 1c because we have no independent measurement of the inductance change caused by the tuning disc. The theory of coupled resonators, however, makes clear predictions for $f_{\mathrm{splitting}}$ as a function of the frequency of either the upper or lower branch. In order to obtain the theoretical curves in Figs. 4, 5 and 6, we have used the standard expression for the impedance of a transmission line terminated in a complex load[24] to derive the response of the complete circuit shown in Fig. 1d.

In Fig. 4 we show $f_{\mathrm{splitting}}$ versus the lower resonant frequency for the case where the tuning disc is above the resonator having the lower initial resonant frequency (Resonator 1 in Fig. 1c). As we pass through the avoided crossing, the splitting initially drops and then rises sharply as the lower resonant frequency tends to a constant value. We see that the data is well fitted by the modelling and that the minimum value of $f_{\mathrm{splitting}}$ is clearly defined. Two variable parameters were used in all the fits: the capacitance per unit length, C and the coupling capacitance C_{k}. The fitted value of C was 6% higher than the theoretical value[25] for a substrate dielectric constant of 10, which is within our calculational error. C_{k} was 34% greater than the COMSOL modelled value. We can obtain more insight into this difference in C_{k} by looking at the other tuned samples.

In Fig. 5 we show the splitting for the 5 samples tested as a function of the coupling capacitance C_{k}. Three of these were selected for tuning experiments. The effect of tuning is to bring the splittings of the three pairs of resonators close to, but not into coincidence with, the simulated values. We note that the difference is not one of scaling, but is primarily an offset $\sim$ 2 MHz. This implies a residual coupling between the resonators which becomes important as the overlap length is reduced and is the origin of the increase in C_{k} required in the fit of Fig. 4. This small residual coupling may well result from inductive coupling between the resonators, or electromagnetic modes in the silicon substrate, which have not been taken into account in our simulations.

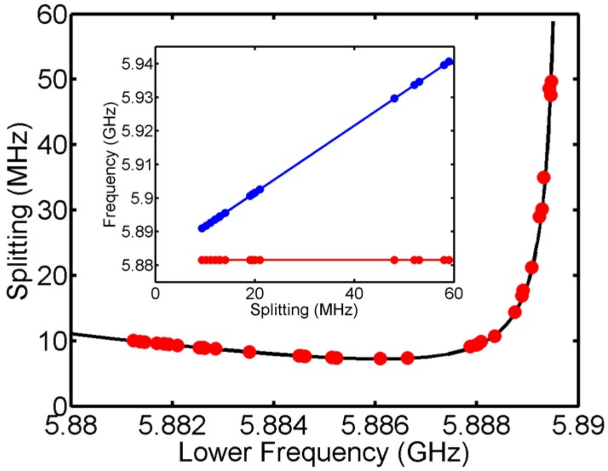

FIG. 4. Splitting between the upper and lower frequency branches of sample 4 as a function of the frequency of the lower branch for the tuning disc above the resonator having the lower *untuned* resonant frequency. Lowering the disc decreases the resonator inductance and takes the resonators through the middle of the avoided crossing. The solid lines are from the simulations, where we have set the inductance per unit length to the theoretical value[25] 4×10^{-7} H/m and capacitance per unit length to 1.8×10^{-10} F/m. The inset shows the upper and lower resonant peaks as a function of $f_{\text{splitting}}$ when the tuning disc was sited above the resonator with the *higher* frequency using the same fitting parameters. A 50 MHz shift of the tuned resonator is easily obtained while the untuned resonator is essentially unaffected.

We also see that, although the differences between the untuned resonator frequencies can be very variable, $f_{\text{splitting}}$ can be rather well predicted. Our CMRs are currently very close to each other and increasing their separation would be expected to substantially reduce the residual coupling.

B. Measured quality factors

The quality factors Q of the two resonances as we traverse the avoided crossing are more complex than for a single resonator because coupled resonators having different individual Qs have branches which must coalesce to a single value at the centre of the avoided crossing. In Fig. 6 we show experimental quality factors and simulations for sample 4. We are interested in the behaviour well away from the avoided crossing, where the Q values of the two branches more closely approximate those of the individual resonators, so here we have plotted the measured Q for the two branches as a function of the *upper* frequency branch. The rapid change at the lower frequency values reflects the flattening of the upper frequency branch with inverse inductance (Fig. 1c, upper curve). Agreement with simulation is not as good as for the resonant frequency data of Fig. 4 because we have

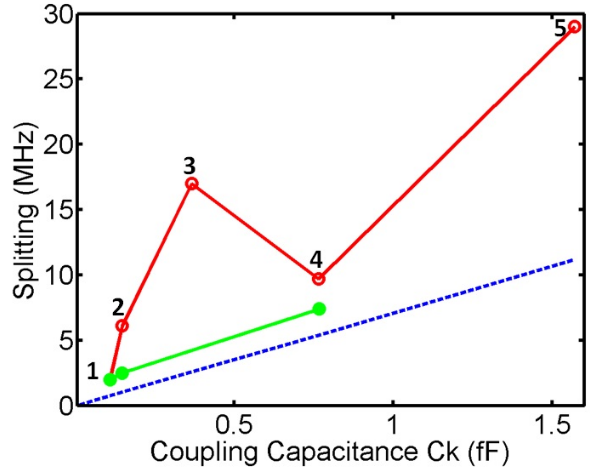

FIG. 5. Splitting versus designed coupling capacitance C_{k} for 5 samples. Three samples were tuned. Open circles show experimental splitting for all samples without tuning; solid circles show the measured splitting at the middle of the avoided crossing for the three tuned samples; the dotted line shows $f_{\text{splitting}}$ expected from the simulations.

less data and it is more scattered, but the general trend is clearly followed. The simplest interpretation is that the tuning disc is not causing any additional degradation of Q within the accuracy of our measurements.

IV. SUMMARY AND CONCLUSIONS

We have demonstrated how two coplanar microwave resonators can be fabricated with a range of coupling strengths close to our design values and how these can be taken controllably through the avoided crossing. The high quality factors do not appear to be degraded by the tuning mechanism. Our most weakly coupled resonator has $f_{\text{splitting}} \sim 1$ MHz, which is well within the range that could be produced with a mechanical system or by driving with oscillating magnetic fields[12,26]. Our observed linewidths are $\sim 10^5$ Hz at 1 K so Autler-Townes splitting and the associated Rabi oscillations should be easily observable in the frequency and time domains respectively and would closely parallel the optical experiments. Our system would provide an excellent test bed for such experiments.

V. ACKNOWLEDGEMENTS

We wish to acknowledge the University of Birmingham for a College Elite Scholarship for B.Gunupudi. We are grateful to Andrew Armour for useful discussions, Alastair Rae for comments on the manuscript and Gary Walsh for technical assistance.

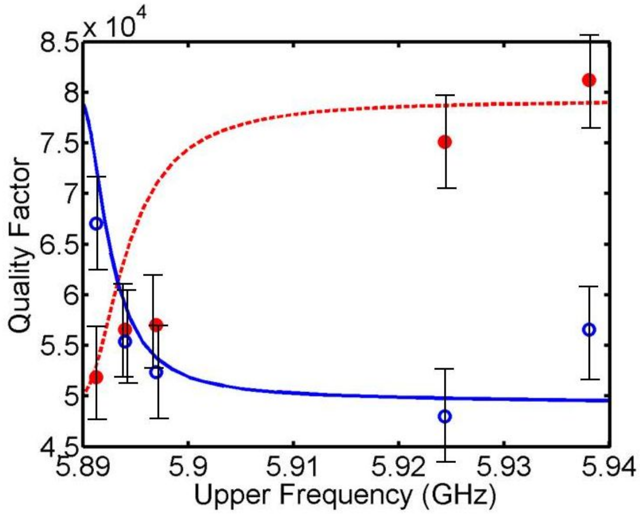

FIG. 6. Experimental and simulated values of loaded Q as a function of the upper resonant frequency. The dashed line and open circles are for the lower resonant frequency branch and the solid line and solid circles are for the upper resonant frequency branch. The values of resonator lengths, C_c, C_k, and the capacitance and inductance per unit length are the same as those used in Fig. 4. The additional parameters used to fit the Qs are the resistance per unit length ρ_1 and ρ_2 of the two resonators. Note that far from the avoided crossing, the sections of the two curves below the crossover can be identified with the resonator being tuned and above the crossing with the untuned resonator.

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